Handout for Week 6: Logic I

Philosophy of Language.
Metavocabularies of Reason:
Pragmatics, Semantics, and Logic
https://sites.pitt.edu/~rbrandom/Courses

Plan:

- **I)** Recap: Three Senses of Explicit/Implicit
- **II)** What is logic? Logicism and Expressivism
- **III)** Rational Logical Expressivism
- **IV)** Elaboration by Multisuccedent Sequent Calculi
- **V)** The Logical System NM-MS: Nonmonotonic Multisuccedent Logic
- **VI)** Combining Supraclassicality and Nonmonotonicity

1. *Closure* structural conditions on consequence relations:

Monotonicity (Tarski): $X \subseteq Y \Rightarrow Con(X) \subseteq Con(Y)$.

Transitivity (Tarski): $Con(Con(X)) = Con(X)$.

Cautious Monotonicity (CM): $\dfrac{X|\!\sim A \quad X|\!\sim B}{X,A|\!\sim B.}$

Cumulative Transitivity (CT): $\dfrac{X|\!\sim A \quad X,A|\!\sim B}{X|\!\sim B.}$ (also called "Shared Context Cut")

2. 3 Senses of Explicit/Implicit

Rationally (consequentially) Explicit/Implicit:

G is part of Γs *explicit* content iff $G \in \Gamma$.

A is part of Γs *implicit* content iff $\Gamma|\!\sim A$.

Rational Explicitation: $\Gamma |\!\sim A \Rightarrow \Gamma,A |\!\sim$?

CM says explicitation never *subtracts* implicit content.

CT says explicitation never *adds* implicit content.

CM+CT say *explicitation is inconsequential*.

Pragmatically Explicit/Implicit:

Implication: X|~A iff explicit *commitment to accept* all of X precludes *entitlement to reject* A.
If one is *explicitly* precluded from *entitlement to reject* A, one is *implicitly committed to accept* A.

A consequence that is *implied* by a premise-set, and so is *rationally implicit* in it, is what one is *implicitly* committed to accept by being *explicitly* committed to accept the premise-set.

Logically Explicit/Implicit:

A *sentence* is what can stand to other sentences in reason relations of implication and incompatibility, both as premise and as conclusion, according to some vocabulary.

According to our pragmatic account of discursive practices, such sentences express *claimables*, what can be accepted or rejected (practical doxastic attitudes), asserted or denied (speech acts), what one can be committed or entitled to (normative deontic statuses).
Logical vocabulary makes reason relations explicit in the sense of expressing those reason relations in the form of claimable sentences.

3. What is logic?

I think the two most important subquestions are:

a) Demarcation Question: What is the distinctive role characteristic of specifically *logical* vocabulary and the concepts such vocabulary expresses?
b) Reasons Question: What is the relation between *logic* and *reasons*?

Two approaches to the Reasons Question:

- **Logicism** about Reasons (LR): *Good* reasons are always *logically* good reasons. Logic determines what reasons are good and explains why they are good.
- Rational **Expressivism** about Logic (REL): Logic expresses reason relations, making them explicit in claimable sentences that can serve as and stand in need of reasons themselves. Logic is an expressive tool for talking about reasons.

4. Expressivism: Logical vocabulary should be **LX** for a prelogical material base vocabulary: conservatively *el*aborated from it (L) and *ex*plicative of it (X), in the sense of making it possible to *make explicit* the reason relations of the base vocabulary by formulating sentences in the logically extended vocabulary that *say that* implications and incompatibilities hold in the base vocabulary.
Further, a fully adequate logical vocabulary should be *universally* LX: able to codify the reason relations of any vocabulary whatsoever, regardless of what structural principles it satisfies.

5. A *vocabulary* $=_{df.}$ A *lexicon* (set of sentences) + *reason relations* on that lexicon.
V = <L, |~, #>. Later: a vocabulary is a lexicon plus a set of pairs of subsets of the lexicon.

6. Three further subsidiary *desiderata* or criteria of adequacy on the relations between a base vocabulary and logical vocabularies that are LX for that base vocabulary:
Three subsidiary *desiderata* or criteria of adequacy will also be motivated:

i) *Universality* of LX-ness: The logical vocabulary should be LX for *every* vocabulary. In particular, it should not be restricted to being LX only for vocabularies that satisfy structural closure principles of monotonicity and transitivity of any grade.
ii) *Conservativeness* of elaboration: the lexicon and reason relations of the base vocabulary (from which the logical vocabulary is elaborated and of which it is explicative) should be contained as subsets in the lexicon and reason relations of the logically extended vocabulary.

iii) *Comprehensiveness* of explicitation: The logical vocabulary should be capable of explicating the reason relations not only of the original base vocabulary, but also the reason relations of the logically extended vocabulary.

7. Elaboration requires a *function* that takes base vocabularies as arguments, and yields logically extended vocabularies as unique values. Since vocabularies consist of lexicons plus reason relations, this means we need two kinds of component functions:

a) Lexical-syntactic, taking the base lexicon as argument and producing the logically extended lexicon as value.
b) Reason-relational, taking the reason relations of the base vocabulary as arguments, and yielding reason relations defined on the logically extended lexicon as values.

8. Conditionals and Negation:

Deduction-Detachment (DD) Condition on Conditionals: $\Gamma |\!\sim A \rightarrow B$ iff $\Gamma, A |\!\sim B$.

Incoherence-Incompatibility (II) Condition on Negation: $\Gamma |\!\sim \neg A$ iff $\Gamma \# A$.

9. Defining the lexical-syntactic elaboration function f_{lex}:

$f_{lex}(L_0) = L$ is the smallest set (by inclusion) such that:

$L_0 \subseteq L$, and if $\alpha, \beta \in L$, then $\alpha \rightarrow \beta$, $\alpha \& \beta$, $\alpha \vee \beta$, and $\neg\alpha$ are $\in L$.

10. Multisuccedent form of reason relations: a set R^2 of pairs of sets $<\Gamma, \Delta>$ s.t: $<\Gamma, \Delta> \in R^2$ iff $\Gamma |\!\sim \Delta$.

This encodes incompatibilities because an empty right-hand side of the turnstile means the premise-set on the left-hand side is incoherent. So $\Gamma, A |\!\sim$ (coded as $<\Gamma \cup \{A\}, \varnothing> \in R^2>$) says that $\Gamma \# A$.

11. Axiom of NM-MS:

$$\frac{\Gamma |\!\sim_0 \Delta}{\Gamma |\!\sim \Delta}$$

($\Gamma |\!\sim_0 \Delta$ can only hold if $\Gamma, \Delta \subseteq L_0$.)

Effect: all the reason relations of the base vocabulary are incorporated into the reason relations of the logically extended vocabulary. So the elaboration is conservative.

12. Example (defined on a logically extended vocabulary whose lexicon is just the base lexicon closed under conjunction):

L&: $$\frac{\Gamma, A, B |\!\sim \Theta}{\Gamma, A \& B |\!\sim \Theta}$$ R&: $$\frac{\Gamma |\!\sim A, \Theta \qquad \Gamma |\!\sim B, \Theta}{\Gamma |\!\sim A \& B, \Theta}$$

1) To define $f_{rat}(R^2_0) = R^2$, the elaboration function for reason relations, we define R^2 as the smallest (by inclusion) set such that:

1) Axiom: $\langle\Gamma,\Delta\rangle \in R^2_0 \;\Rightarrow\; \langle\Gamma,\Delta\rangle \in R^2$
2) L&: $\langle\Gamma\cup\{A\}\cup\{B\}, \Theta\rangle \in R^2 \;\Rightarrow\; \langle\Gamma\cup\{A\&B\}, \Theta\rangle \in R^2$
3) R&: $(\langle\Gamma,\{A\}\cup\Theta\rangle \in R^2$ and $\langle\Gamma,\{B\}\cup\Theta\rangle \in R^2) \Rightarrow \langle\Gamma, \{A\&B\}\cup\Theta\rangle \in R^2$

These three rules tell us:

(1) Which sequents (pairs of sets of sentences, representing premises and conclusions of reason relations) from the base vocabulary are in the &-extended vocabulary—all of them. And
(2) Which sequents containing conjunctions in their premise-sets are in the &-extended vocabulary. And
(3) Which sequents containing conjunctions in their conclusion-sets are in the &-extended vocabulary.

We are going to do the same thing with all the connective rules of NM-MS.
The only thing that changes with more connectives is that we have to make R^2 the smallest set that includes the result of applying *all* the rules to the base vocabulary $\langle L_0, R^2_0\rangle$.

13. Rules of **N**on-**M**onotonic **M**ulti-**S**uccedent (NM-MS) Sequent Calculus:

Axioms:
For every sequent $\Gamma |\sim_0 \Theta$ in the base, $\Gamma |\sim \Theta$ may be assumed as an axiom (base of proof tree).

Connective Rules (Ketonen):

L→: $$\frac{\Gamma |\sim \Theta, A \qquad B,\Gamma |\sim \Theta}{A \to B,\Gamma |\sim \Theta}$$

R→: $$\frac{A,\Gamma |\sim \Theta, B}{\Gamma |\sim \Theta, A \to B}$$

L&: $$\frac{\Gamma, A, B |\sim \Theta}{\Gamma, A\&B |\sim \Theta}$$

R&: $$\frac{\Gamma |\sim A, \Theta \qquad \Gamma |\sim B, \Theta}{\Gamma |\sim A\&B, \Theta}$$

L∨: $$\frac{A,\Gamma |\sim \Theta \qquad B,\Gamma |\sim \Theta}{A\vee B, \Gamma |\sim \Theta}$$

R∨: $$\frac{\Gamma |\sim A, B, \Theta}{\Gamma |\sim A\vee B, \Theta}$$

L¬: $$\frac{\Gamma |\sim A, \Theta}{\neg A, \Gamma |\sim \Theta}$$

R¬: $$\frac{A,\Gamma |\sim \Theta}{\Gamma |\sim \neg A, \Theta}$$

14. Compare (LK) on conjunction:

L&(additive): $$\frac{\Gamma, A |\sim \Theta}{\Gamma, A\&B |\sim \Theta} \qquad \frac{\Gamma, B |\sim \Theta}{\Gamma, A\&B |\sim \Theta}$$

This would force monotonicity on the left of the turnstile.
NM-MS has a multiplicative rule.

Similarly, we have a multiplicative right-rule for disjunction.
LK's additive one would force monotonicity on the right.

15. We must distinguish:

i) The reason relations of vocabularies logically elaborated from base vocabularies by NM-MS, and
ii) Purely logical reason relations of NM-MS: those that hold in virtue of logic alone.

There are two things one can mean by "reason relations over the logically extended lexicon that hold in virtue of logic alone":

iii) Local: Within any such specific NM-MS extension of a particular base vocabulary, the consequences that are good and remain good on arbitrary replacement of non-logical with non-logical vocabulary.
iv) Global: The consequences involving logically complex sentences that hold good no matter what base vocabulary we apply NM-MS to.

These two do *not* in general coincide.

16. Supraclassicality:

- NM-MS is supraclassical w/res to *theorems*, in that every theorem of LK is a theorem of NM-MS.
- NM-MS is *not* supraclassical w/res to *consequence relations*. Not every metainference licensed by LK is licensed by NM-MS

For all premise-sets Γ: Γ |~ (A→C)→((A&B)→C) (a classical theorem).
Why doesn't that force MO:

$$\frac{\Gamma,A|\!\sim C}{\Gamma,A,B|\!\sim C}\ ?$$

Claim 1: Can detach from conditionals on the left (both conditional and antecedent explicit).
Γ, A, A→B |~ B.
L→ (which, recall, is reversible) says that to get Γ,A, A→B |~ B, we need two premises:
(In L→, Θ is in our example, B, and (confusingly) Γ is Γ,A.
Γ,A |~ B,A and Γ,A,B |~ B.
But both of these are instances of CO, so we always have them.

$$\frac{\mathbf{\Gamma,A |\!\sim B,A \quad and\ \Gamma,A,B |\!\sim B}}{\mathbf{\Gamma,A,A\to B |\!\sim B.}}$$

Claim 2: Can*not* detach from conditionals on the right (both conditional and antecedent implicit).
We can*not* argue from Γ |~ A and Γ |~ A→B to Γ |~ B.
That is: **NOT**: $\dfrac{\Gamma |\!\sim A \qquad \Gamma |\!\sim A\to B}{\Gamma |\!\sim B}$

Why not? Γ |~ A→B iff (reversible R→) Γ,A |~ B.
So the argument we are assessing can be rewritten as:

$$\frac{\Gamma \mid\sim A \qquad \Gamma, A \mid\sim B}{\Gamma \mid\sim B}$$

That is just Cut (CT).

That does not hold in NM-MS.

<u>Claim 3</u>: Can*not* detach an explicit conditional with an implicit (implied) antecedent.

What about Γ|~A and Γ,A→B|~C? Can we get Γ|~C? No.

If Γ,A→B|~C, we know by L→ that Γ|~C,A and Γ,B|~C.

We also have Γ|~A.

So the argument at issue is:

$$\frac{\Gamma \mid\sim A \quad \Gamma \mid\sim C, A \qquad \Gamma, B \mid\sim C}{\Gamma \mid\sim C} \qquad ?$$

The third premise is irrelevant, since B isn't in the conclusion or the other premises.

But what we can get from Γ|~C, A is Γ,¬A|~C, and Γ|~A doesn't help getting Γ|~C.

With these 3 claims about detachment settled, let's look again at how and why

Γ |~ (A→C)→((A&B)→C)

fails to license the monotonicity move:

$$\frac{\Gamma, A \mid\sim C}{\Gamma, A, B \mid\sim C.}$$

R→ tells us that Γ |~ (A→C)→((A&B)→C) must have come from

Γ,A→C |~ (A&B)→C.

That in turn comes from Γ,A→C,A&B |~ C.

Unpacking that by L&, we need to have Γ,A→C,A,B |~ C.

But that we get by detachment on the left.

That is why Γ |~ (A→C)→((A&B)→C) for every Γ.

The implication Γ,A,A→C |~ C can be weakened by arbitrary B.

But that does not at all imply that Γ,A |~ C can be weakened by arbitrary B, which is the monotonicity move.

It is essential to the goodness of Γ,A,A→C,B |~ C that the conditional is in the antecedent.